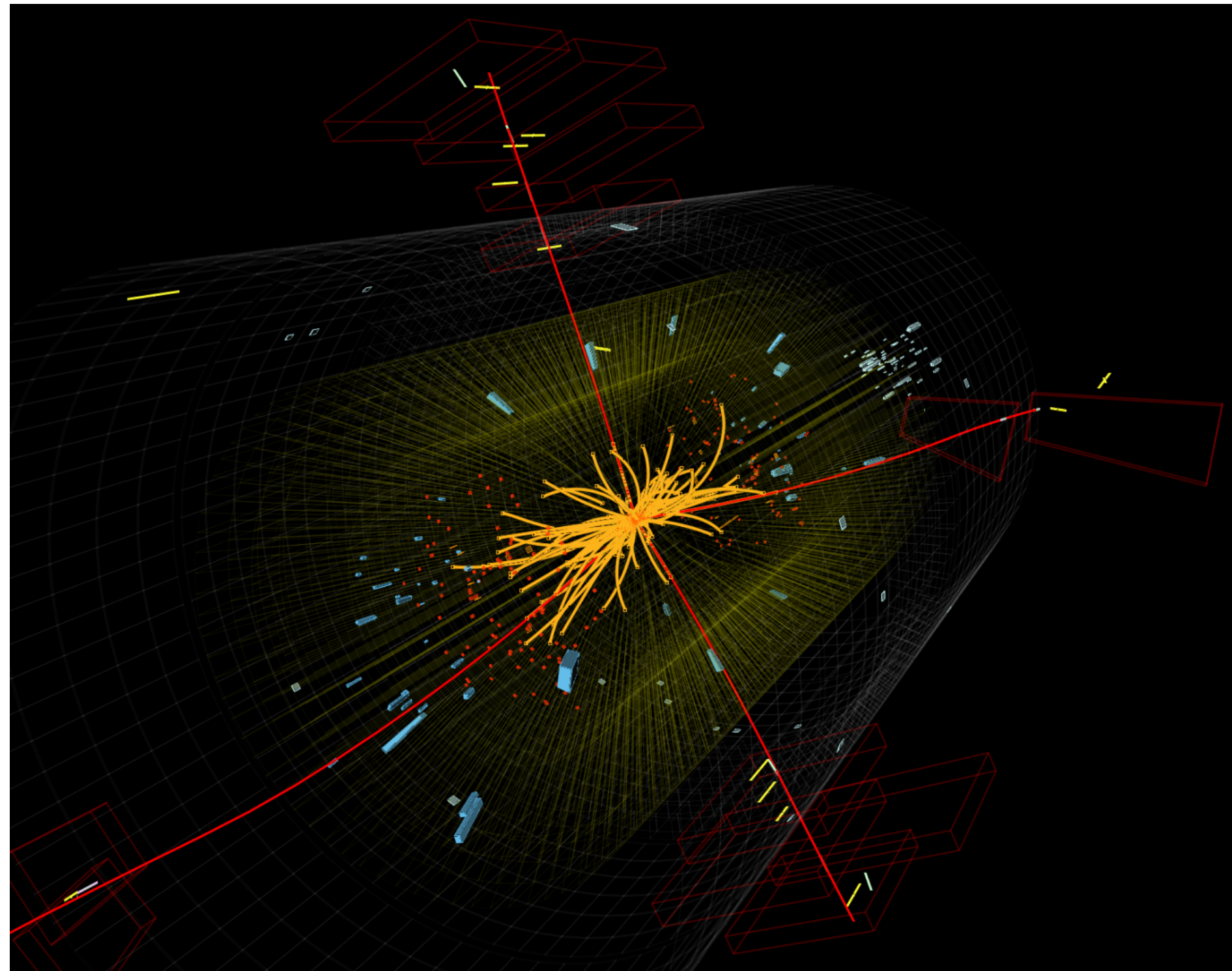


Machine learning applications at high-energy particle colliders to probe triple Higgs coupling



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arXiv:22XX.XXXXX

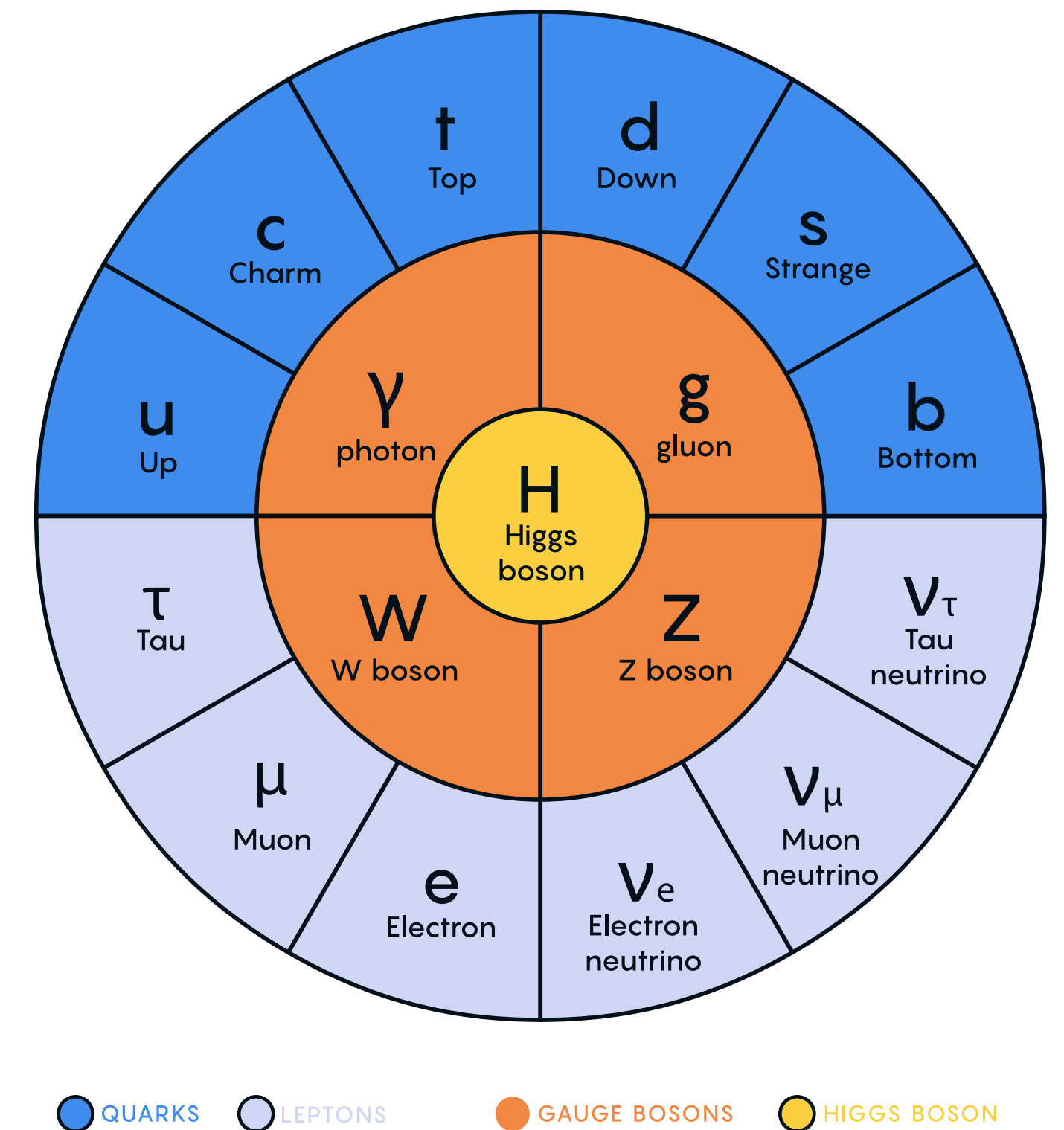
Workshop for AI-Powered Materials Discovery at Great Plains June 22-25, 2025
University of South Dakota

Standard model of particle physics

- A quantum field theory that describes electromagnetic, weak and strong interactions

$$\text{Gauge group : } \underbrace{SU(3)_C}_{\text{QCD}} \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{Electroweak}}$$

- Particle content : Quarks (spin-1/2), leptons (spin-1/2) and gauge bosons (spin-1 and 0)
- Successful in predicting masses of W, Z, leptons, quarks (bare mass)
- Most parameters measured with high accuracy except triple-Higgs coupling



Triple Higgs coupling

- First interaction term in Higgs potential

$$V(h) = -\mu^2\phi^2 + \lambda\phi^4 \equiv \frac{1}{2}m_h^2h^2 + \lambda_{hhh}h^3 + \lambda_{hhhh}h^4$$

Triple Higgs coupling

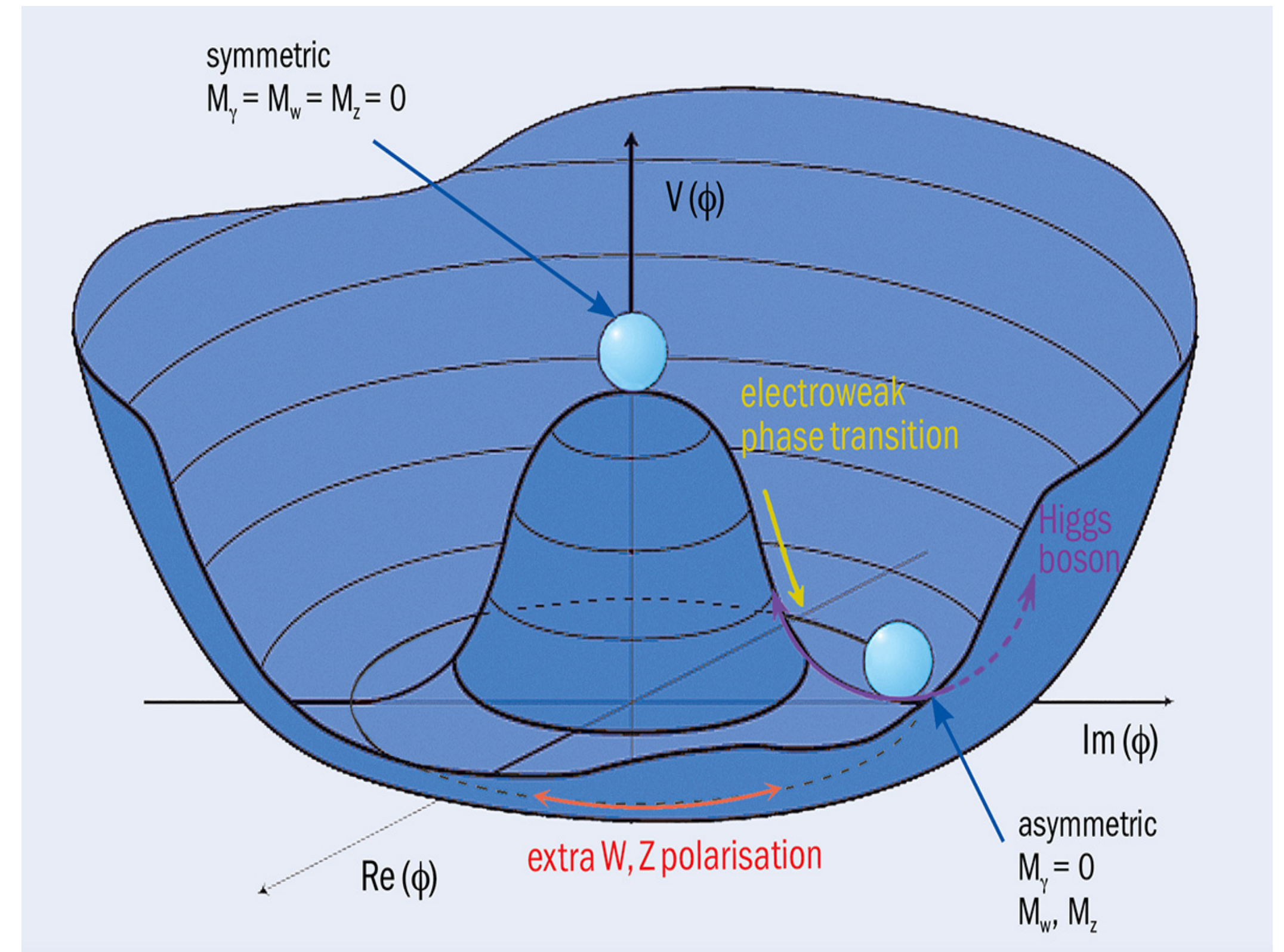
Quartic self coupling

1. Important to complete SM
2. Important to constraint any physics beyond SM
3. Determines electroweak phase transition in early universe

$$m_h = 125 \text{ GeV}$$

$$\lambda_{hhh} = \lambda_{hhhh} v$$

$$\lambda_{hhhh} = \frac{m_h^2}{2v^2}$$

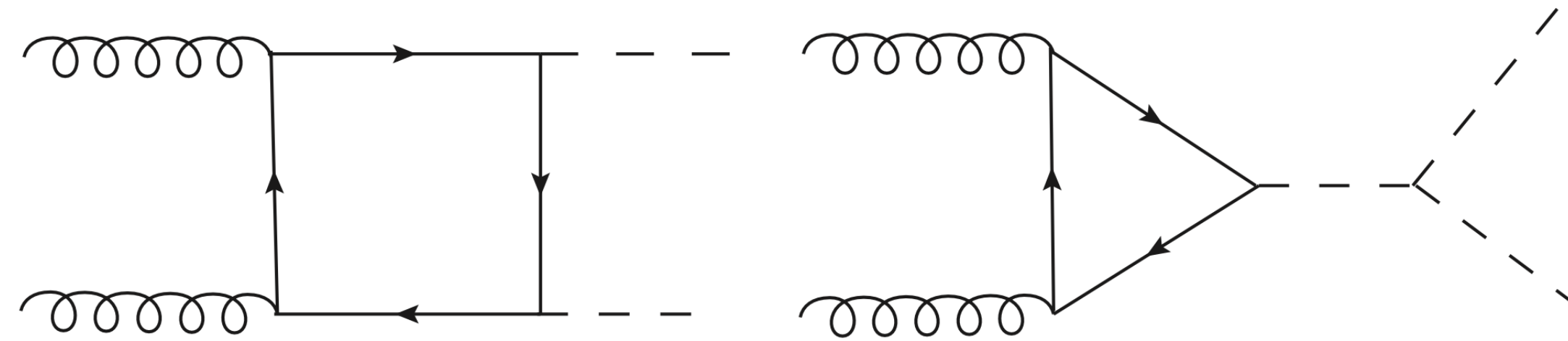


Experimentally accessible through double Higgs and single Higgs production at high-energy particle colliders such as proton-proton (pp), electron-positron

Hadron colliders

- Direct probe for triple Higgs coupling is $gg \rightarrow HH$

λ_{3H} at leading order (LO) $\longrightarrow$ direct channel



NPB 309 (1988) 282

Accuracy of the determination
of λ_{3H} is low

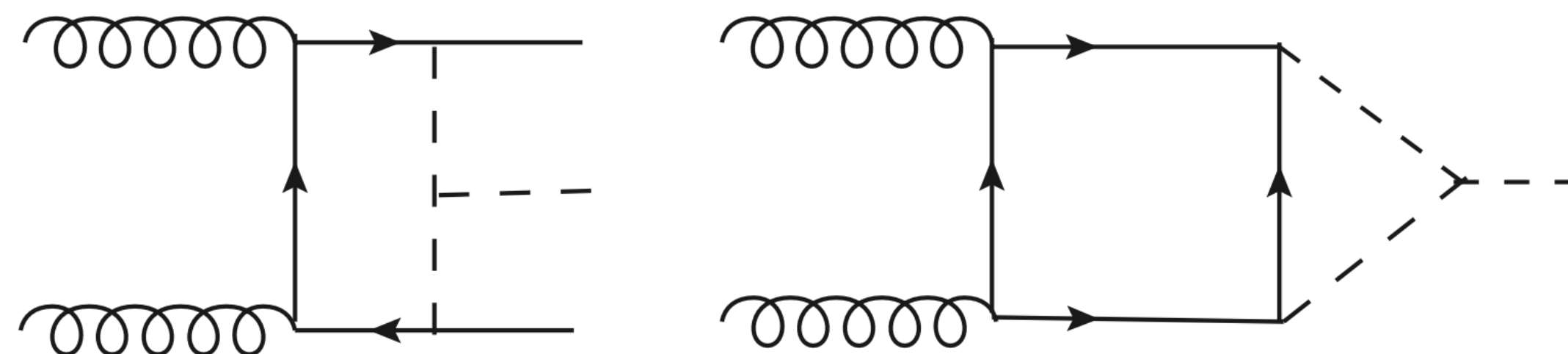
- Cancellation between box and triangle diagrams: **destructive interference**
- Destructive interference leads to extremely small cross-section
- High beam energy is required
- Indirect channel: λ_{3H} enters at NLO

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1702.07678

HL-LHC

$$-0.8 < \frac{\lambda_{3H}}{\lambda_{SM}} < 7.7$$

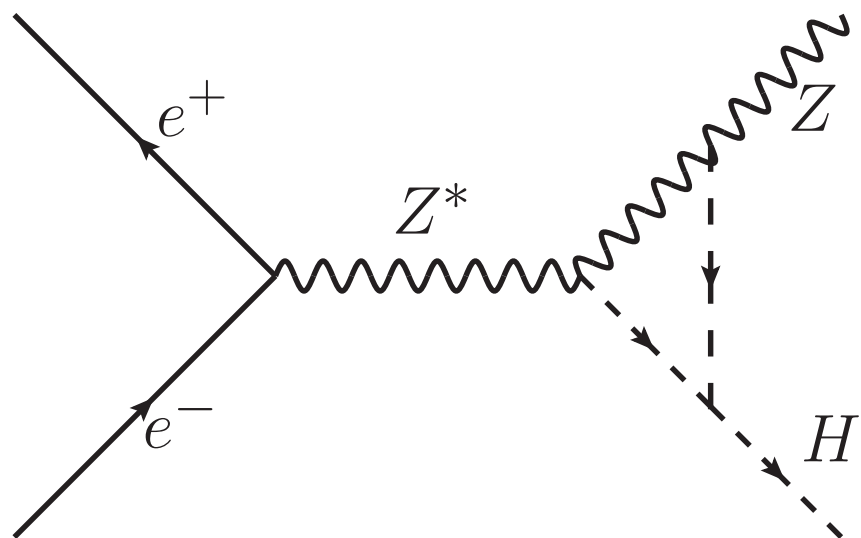
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Electron positron colliders

- Higgs associated production is sensitive to λ_{3H} at NLO in e^+e^- collider: Indirect measurement

1312.3322



$$\lambda_{3H} = \lambda_{SM}(1 + \delta_h)$$

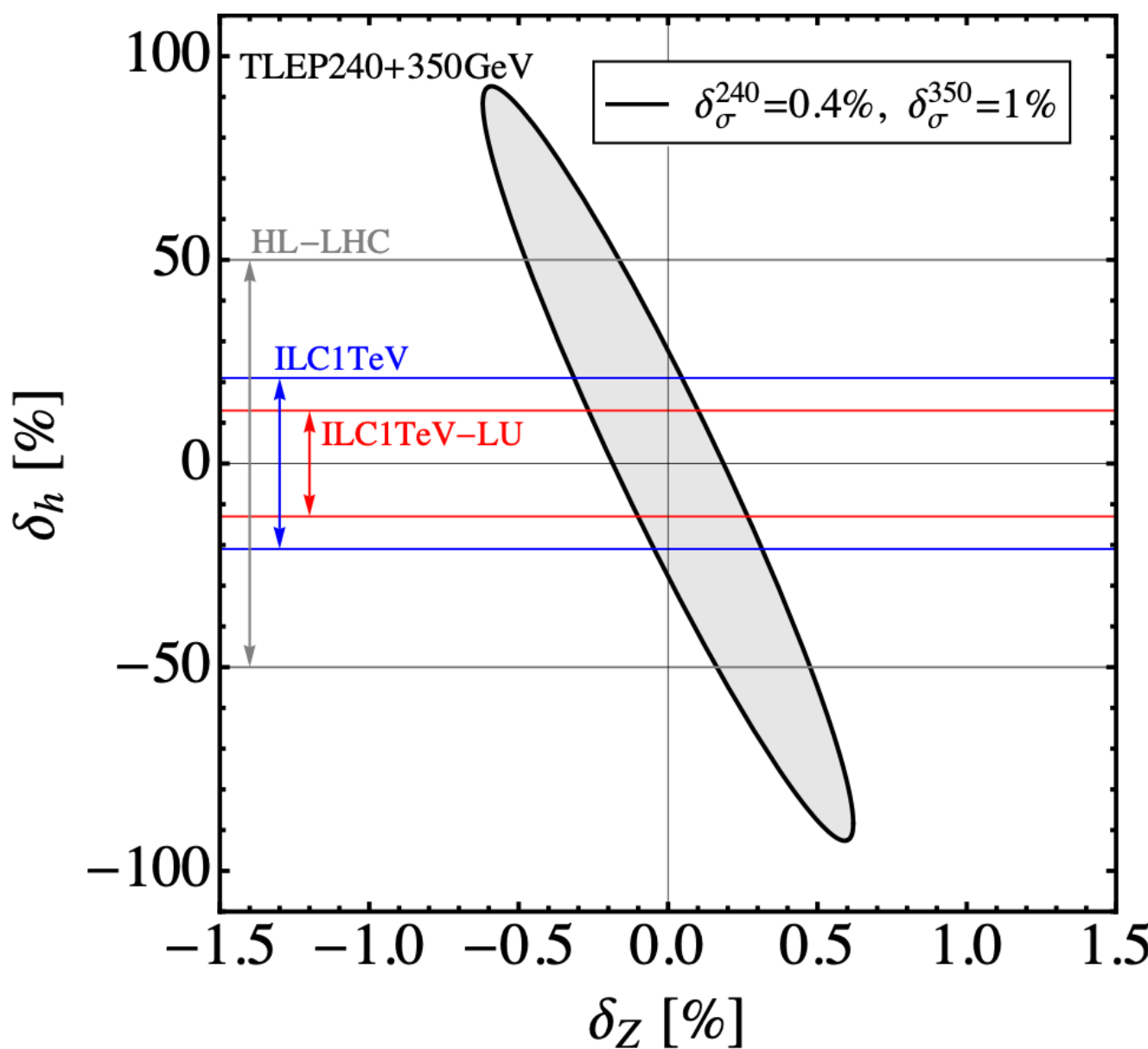
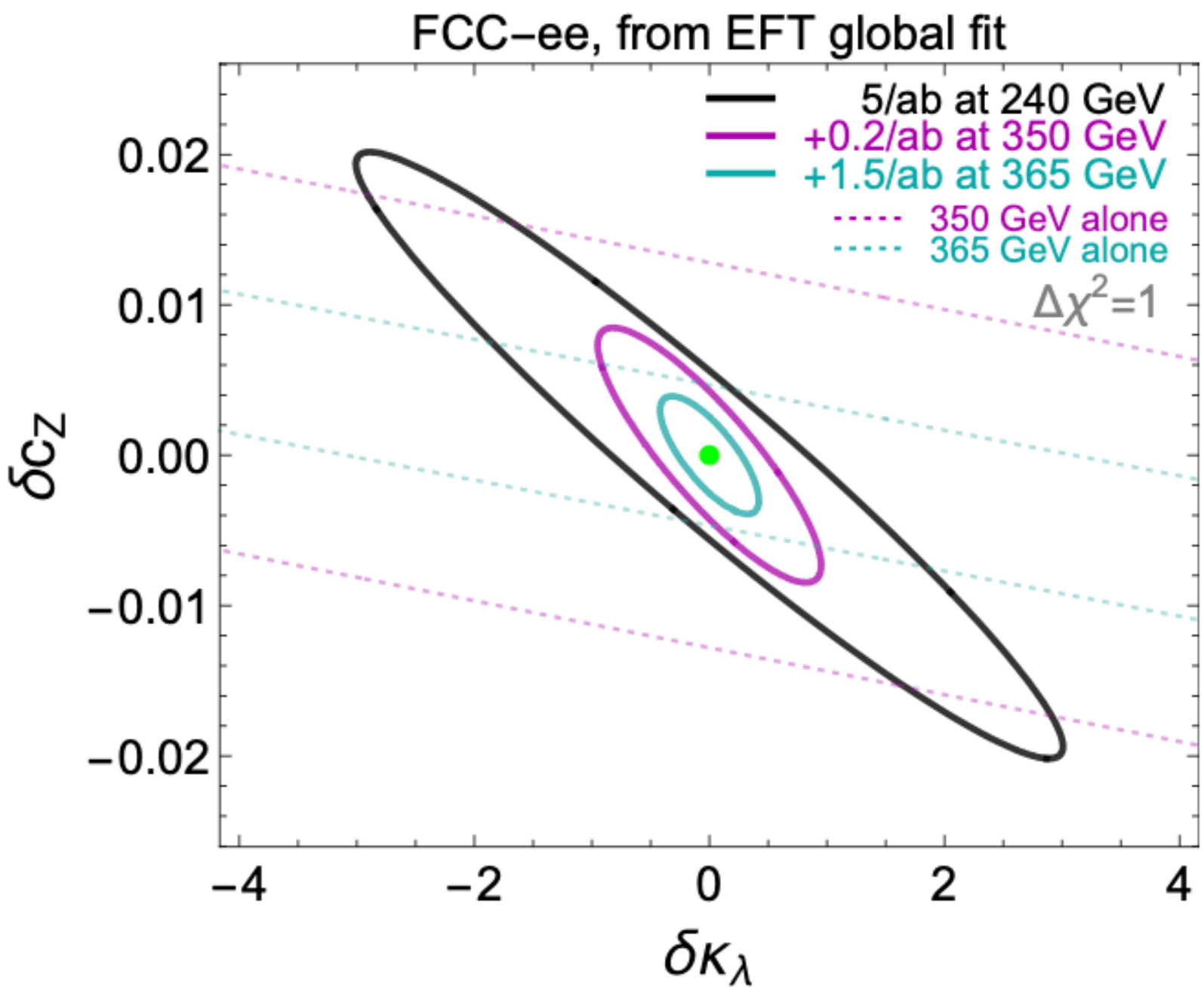
δ_h Parameterises deviation to self coupling

Can constrain δ_h in a model dependent way $\longrightarrow$ FCC-ee at $\sqrt{s} = 240$ GeV by 28%

$$\mathcal{L} \sim 10 \text{ ab}^{-1}$$

1809.10041

- With HZZ (δ_Z), combination of δ_Z and δ_h can be constrained
- Precision measurements at different c.o.m energies can be useful to constrain the combination of δ_h and δ_Z



Sensitivity can be improved if δ_Z can be removed somehow!

1704.01953

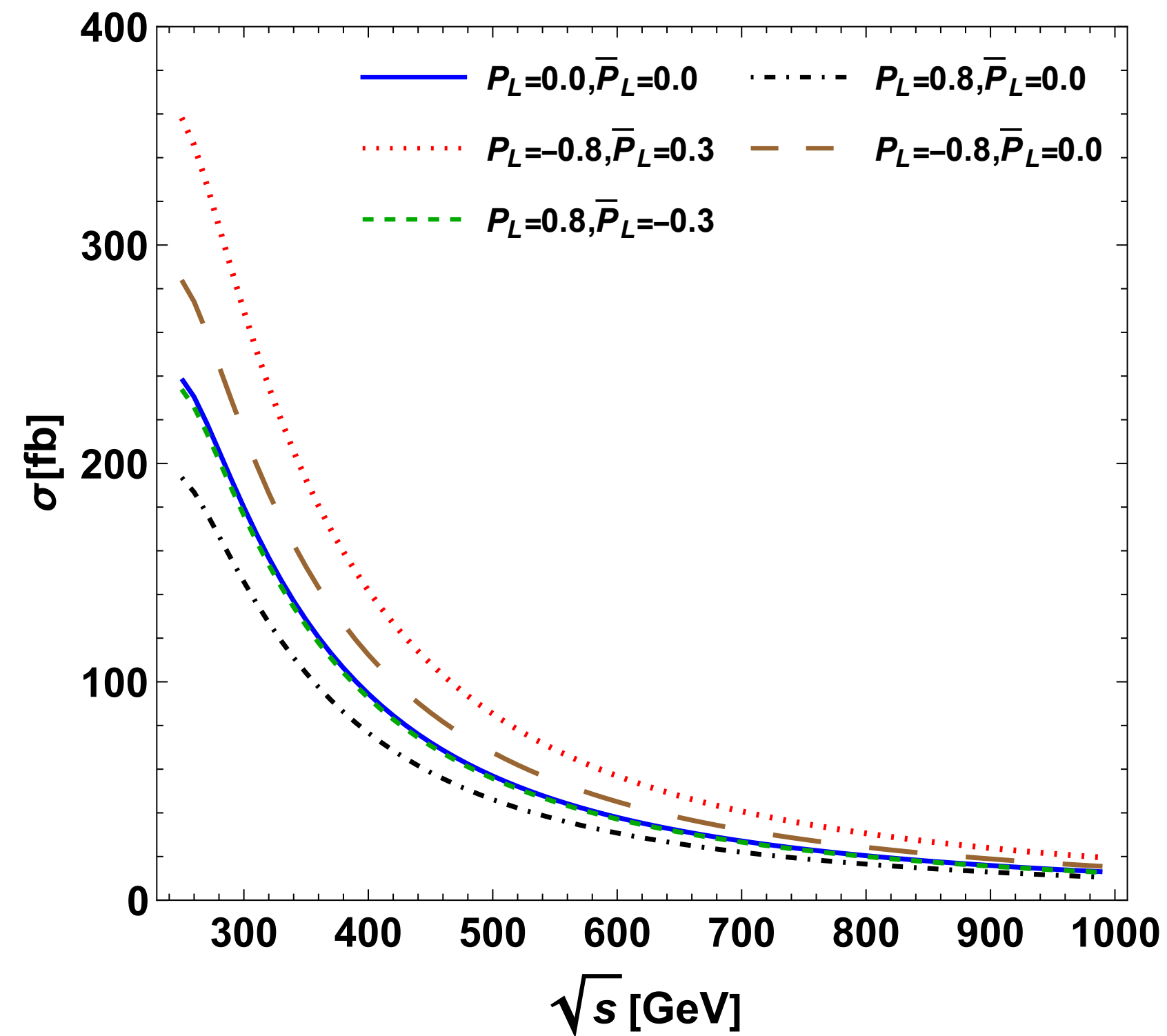
Trilinear coupling at $e^+e^- \rightarrow ZH$

- Fractional change in cross-section and 1σ limit on κ
- Polarized beam can improve sensitivity

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$\mathcal{L} \sim 2 \text{ ab}^{-1}$



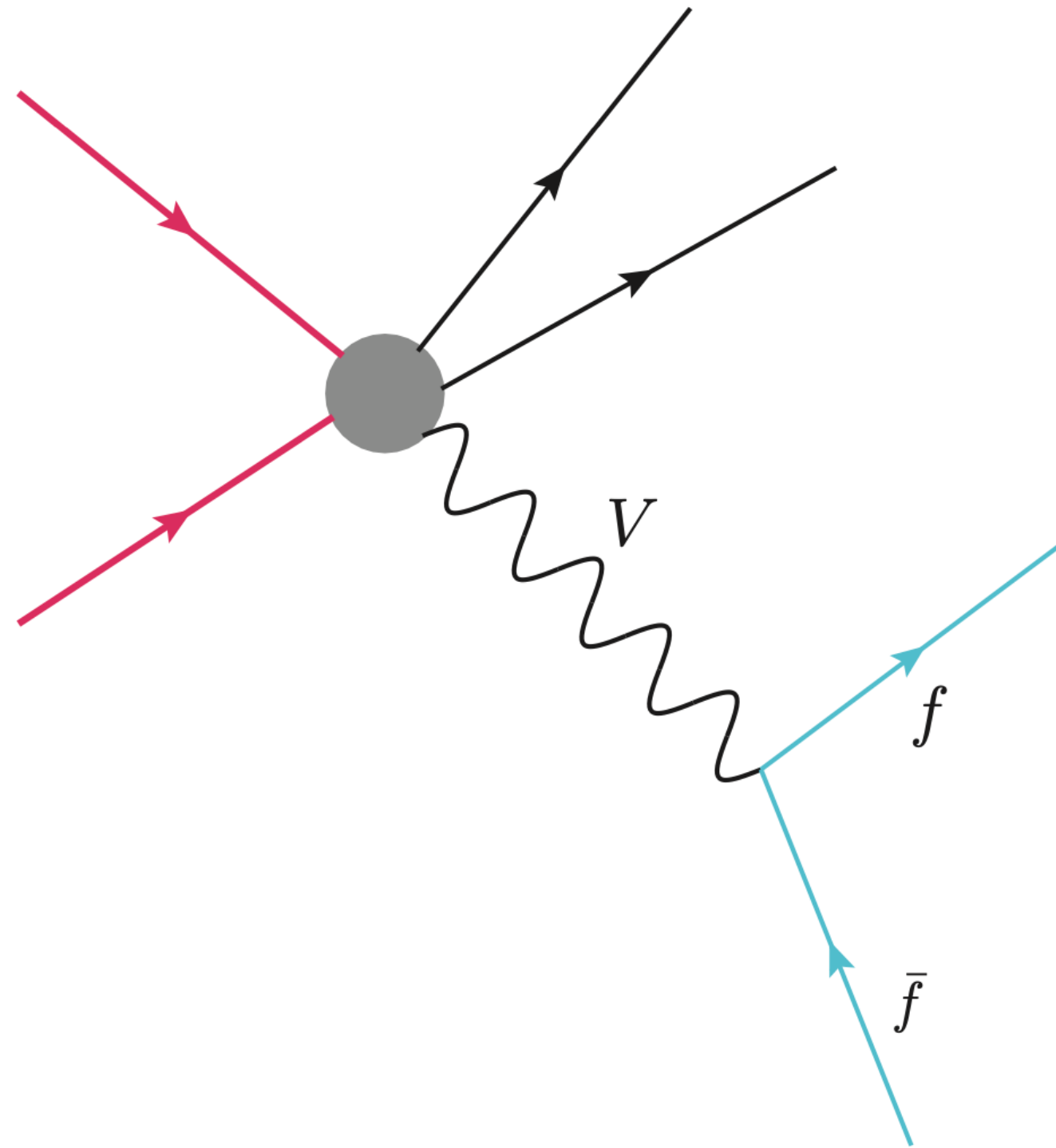
$\sqrt{s}$	P_L	$\bar{P}_L$	σ_L (fb)	$\frac{\delta\sigma}{\sigma}/\kappa$	κ_{lim} (%)
250	0	0	242	1.278	70.0
	-0.8	0	288	1.278	64.2
	-0.8	+0.3	364	1.278	57.2
350	0	0	129	0.284	315
	-0.8	0	153	0.284	289
	-0.8	+0.3	193	0.284	257
500	0	0	56.9	-0.203	-440
	-0.8	0	67.6	-0.203	-403
	-0.8	+0.3	85.3	-0.203	-359
1000	0	0	12.7	-0.433	-206
	-0.8	0	15.1	-0.433	-189
	-0.8	+0.3	19.1	-0.433	-169

- For $P_{e^-} = -0.8$ and $P_{e^+} = 0.3$ the accuracy to measure κ is about 57 %

Z polarization for triple Higgs

- Angular asymmetries of decay products

2109.11134



$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega_f} \propto \sum_{\lambda \lambda'} P(\lambda, \lambda') \Gamma(\lambda, \lambda'; \theta, \phi)$$

$$\lambda, \lambda' \in (-1, 0, 1)$$

$P(\lambda, \lambda') \longrightarrow$ Polarization density matrix

$\Gamma(\lambda, \lambda'; \theta, \phi) \longrightarrow$ Decay density matrix

- θ and ϕ are polar and azimuthal angles of final state fermion in the rest frame of V
- Polarization parameters can be extracted from polarized matrix elements

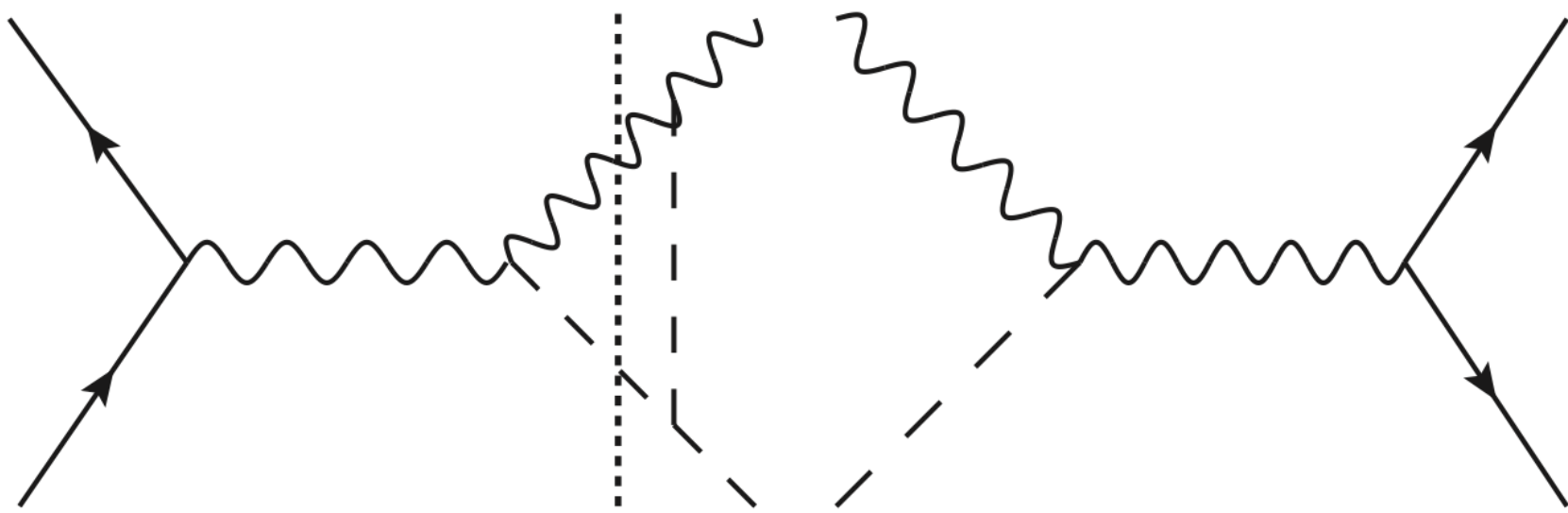
Z polarization for triple Higgs

- Extraction of trilinear coupling from ZH production is overwhelmed by tree level anomalous couplings

Anomalous ZZH coupling: Dimension-six operators in SMEFT

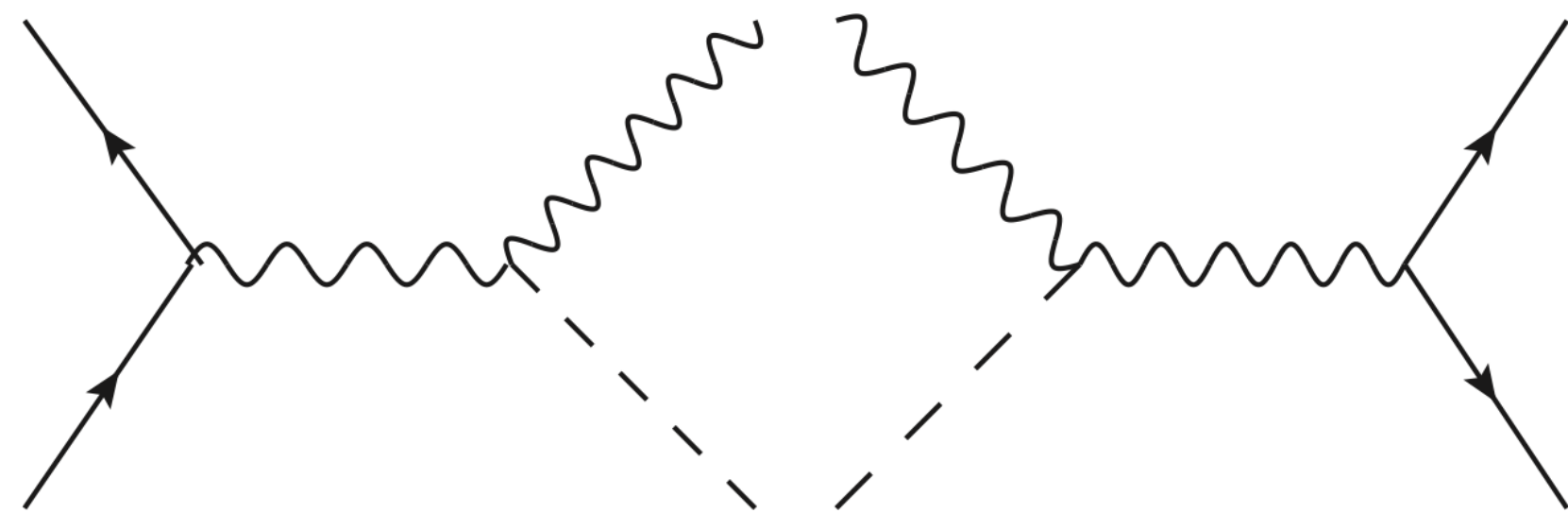
2109.11134

- T-odd distributions of the production cross-section can be used



Tree level contributions are T-even

Can constrain λ_{3H} independent of tree level anomalous couplings



We explore the possibility of using Z polarization for measuring trilinear Higgs coupling

Either less sensitive to tree level ZZH coupling or independent of it

Z polarization for triple Higgs

2109.11134

Collider	c.m. energy (GeV)	$10^4 \times A_{yz}$		Lumi- nosity (ab^{-1})	Limit	
		unpolarized beams	polarized beams		unpolarized beams	polarized beams
CEPC	240	-0.159	-10.6	10	506	31.0
CEPC	240	-0.159		20	358	
CLIC	380	-2.88		0.5	124	
FCC	240	-0.159		10	506	
FCC	250	-0.314		5	362	
FCC	365	-2.64	-1.23	1.5	78.2	119
ILC	250	-0.314		2	573	
ILC	250	-0.314		5	362	
ILC	350	-2.39		30	19.4	
ILC	500	-4.00		4	31.6	
ILC	500	-4.00	-15.7	10	20.0	4.16
ILC	500	-4.00		30	11.5	2.40

Can we improve the sensitivity with machine learning techniques ?

Graph Neural Network (GNN)

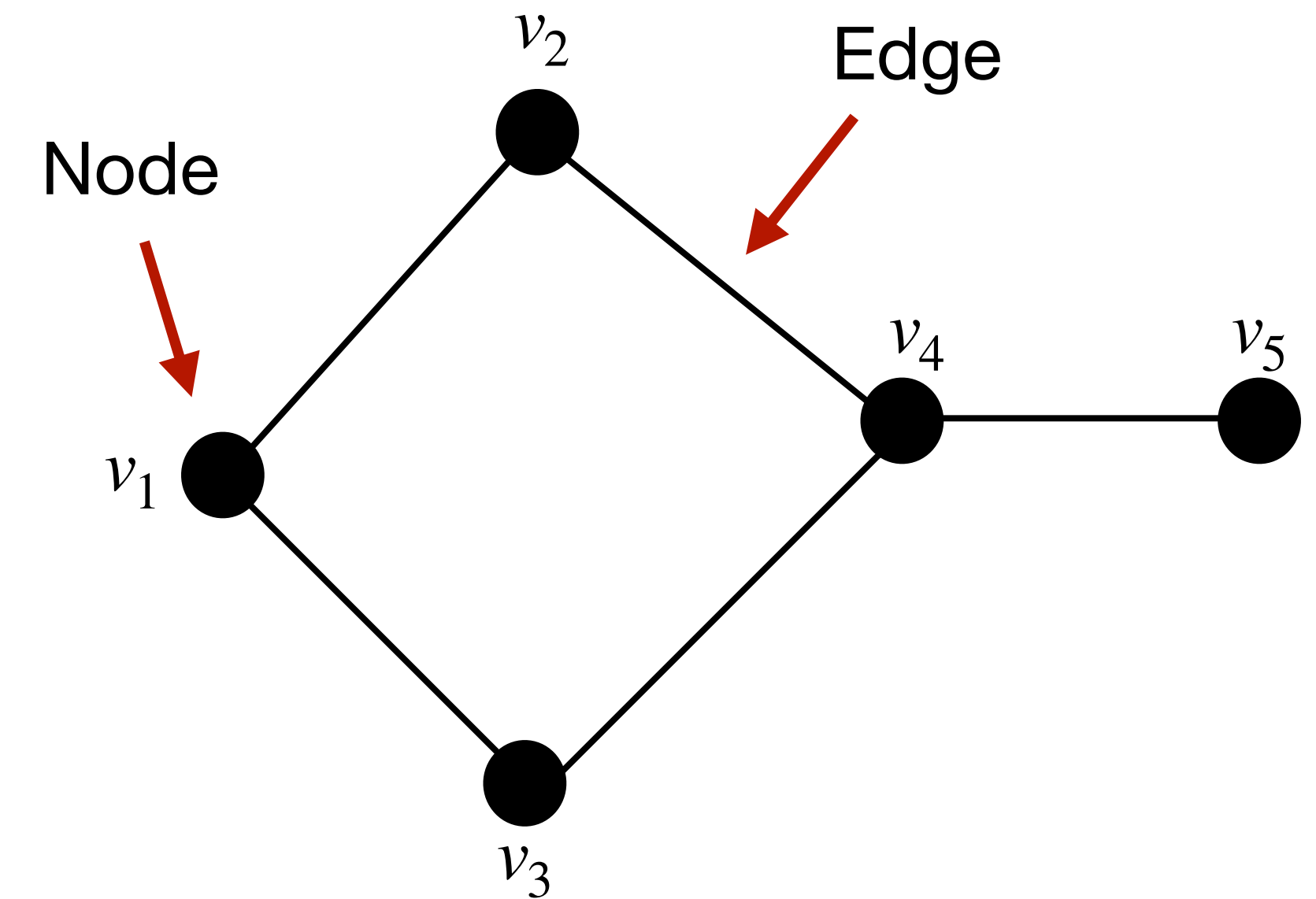
- Graph : A data structure to represent entities and their relationships.

$$G = (V, E) \quad E \subseteq V \times V$$

- GNNs are a class of deep learning models that operate directly on graph structures.
- Learn entire graph-level representations by passing and aggregating information along the edges.

The general update rule:

$$\mathbf{h}_v^{(k)} = \text{UPDATE}^{(k)} \left(\mathbf{h}_v^{(k-1)}, \text{AGGREGATE}^{(k)} \left(\left\{ \mathbf{h}_u^{(k-1)} : u \in \mathcal{N}(v) \right\} \right) \right)$$



Nodes : $V = \{v_1, v_2, v_3, v_4, v_5\}$

Edges : $E = \{(v_1, v_2), (v_1, v_3), (v_2, v_4), (v_3, v_4), (v_4, v_5)\}$

AGGREGATE combines features from neighbors

UPDATE combines aggregated messages

Graph Convolution Network (GCN)

- Generalize convolution neural network to graph structured data

Nodes aggregate information from their neighbors through adjacency structure

GCN layer update rule:

$H^{(l)}$: Node feature matrix at layer l

Learnable weight matrix at layer l

Diagonal matrix: $\hat{D}_{ii} = \sum_j \hat{A}_{ij}$

$$H^{(l+1)} = \sigma \left(\tilde{D}^{-1/2} \tilde{A} \tilde{D}^{-1/2} H^{(l)} W^{(l)} \right)$$

Nonlinear activation ReLU

$\tilde{A} = A + I$: Adjacency matrix

$H^{(0)}$ = input features

Graph Convolution Network tasks

1. Node level prediction

Identification of new particle

Classify scientific paper into categories

2. Graph level prediction

Classify an event as signal or background in particle colliders

Classify a jet as originating from top quark, W/Z/H boson or QCD background

Classify quenched or vacuum like jet in heavy-ion collision

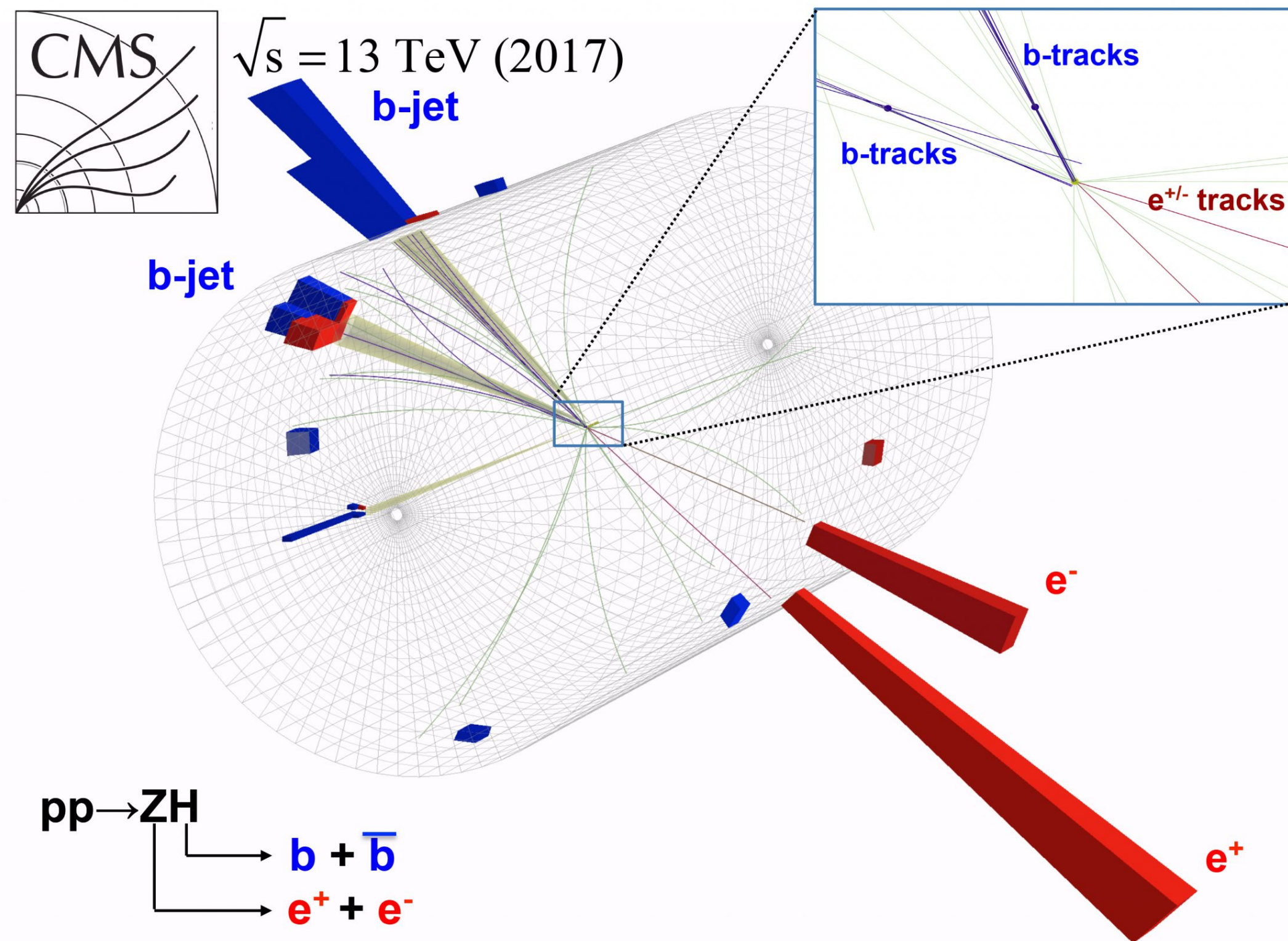
3. Edge level prediction

Jet clusterings

Predict whether two partons are color connected or not

GNN in particle physics

- In particle physics we measure hadrons with certain transverse momentum p_T and energy E



1. Jets are collimated sprays of particles defined by jet algorithms
2. Charged particles measured by tracks
3. b jets are identified with acceptable efficiency $> 80 \%$
4. All final state particles including jets are connected by well defined invariant mass or angular separation between the particles

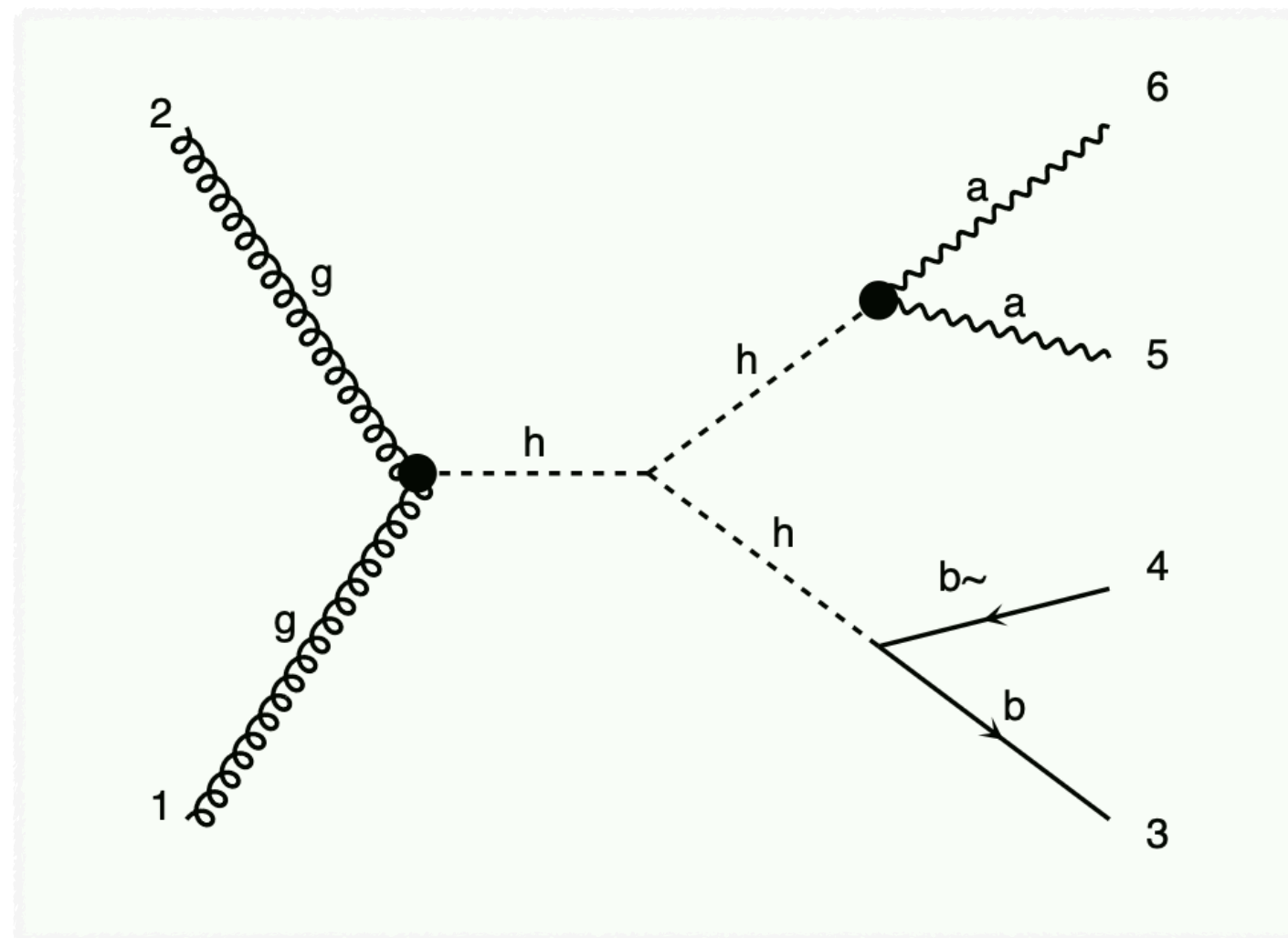
- Full event information including tagging efficiency and missing energy can be used to construct a graph

GNN for triple Higgs coupling : parton level

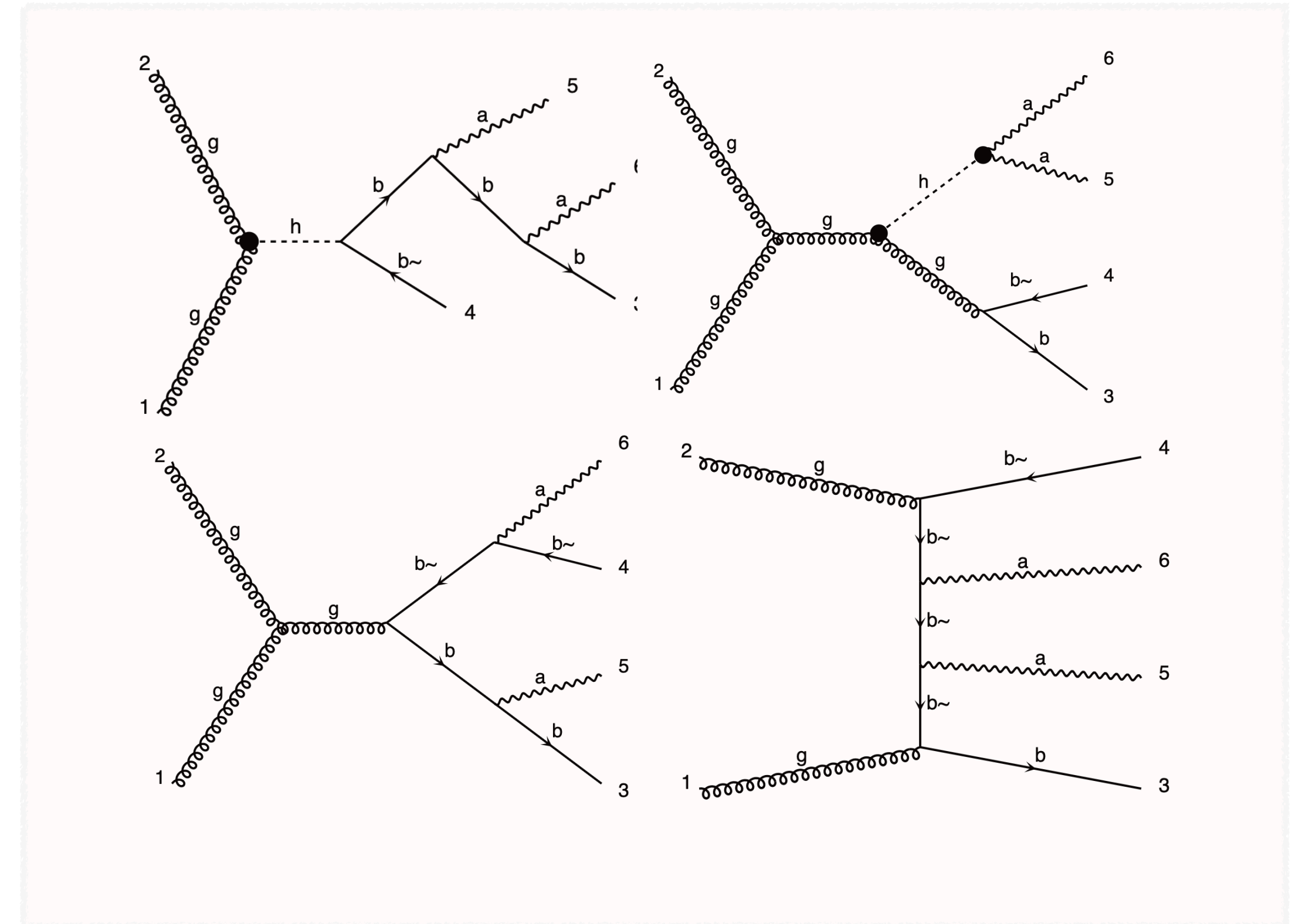
- For a given channel for triple Higgs coupling final state particles define nodes

Channel : $gg \rightarrow HH \rightarrow b\bar{b}\gamma\gamma$

Signal



- Clean signal and less background
- Branching ratio is small



Background processes

Goal : build event-level graph representation for machine learning

Graph preparation

n	b	b~	γ	γ	pT	eta	phi
b	1	0	0	0	44	-0.19	0.34
b~	0	-1	0	0	37	-0.29	-3.05
a	0	0	1	0	25	0.02	-2.98
a	0	0	0	1	22	-0.80	-0.37

Each event is represented by a graph

Nodes : Final state particles

Node features : Four momenta of final state particles

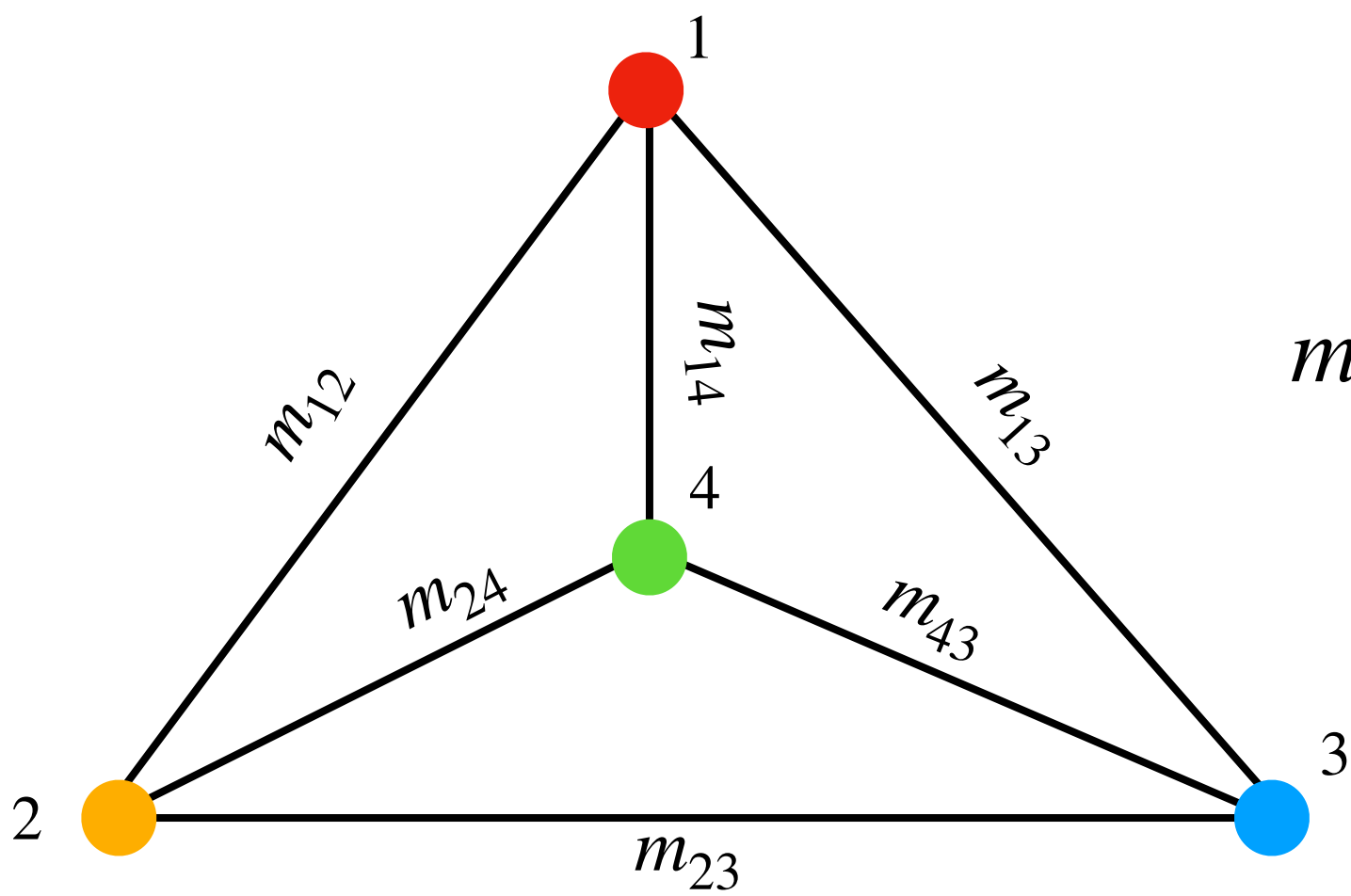
Edges : Invariant mass of final state particles

1. Each event stored as 4×7 dim matrix

$$\text{Shape} = (N_{\text{nodes}}, N_{\text{features}})$$

2. Each node stores 7 dim feature vector

3. Missing energy can be stored in feature



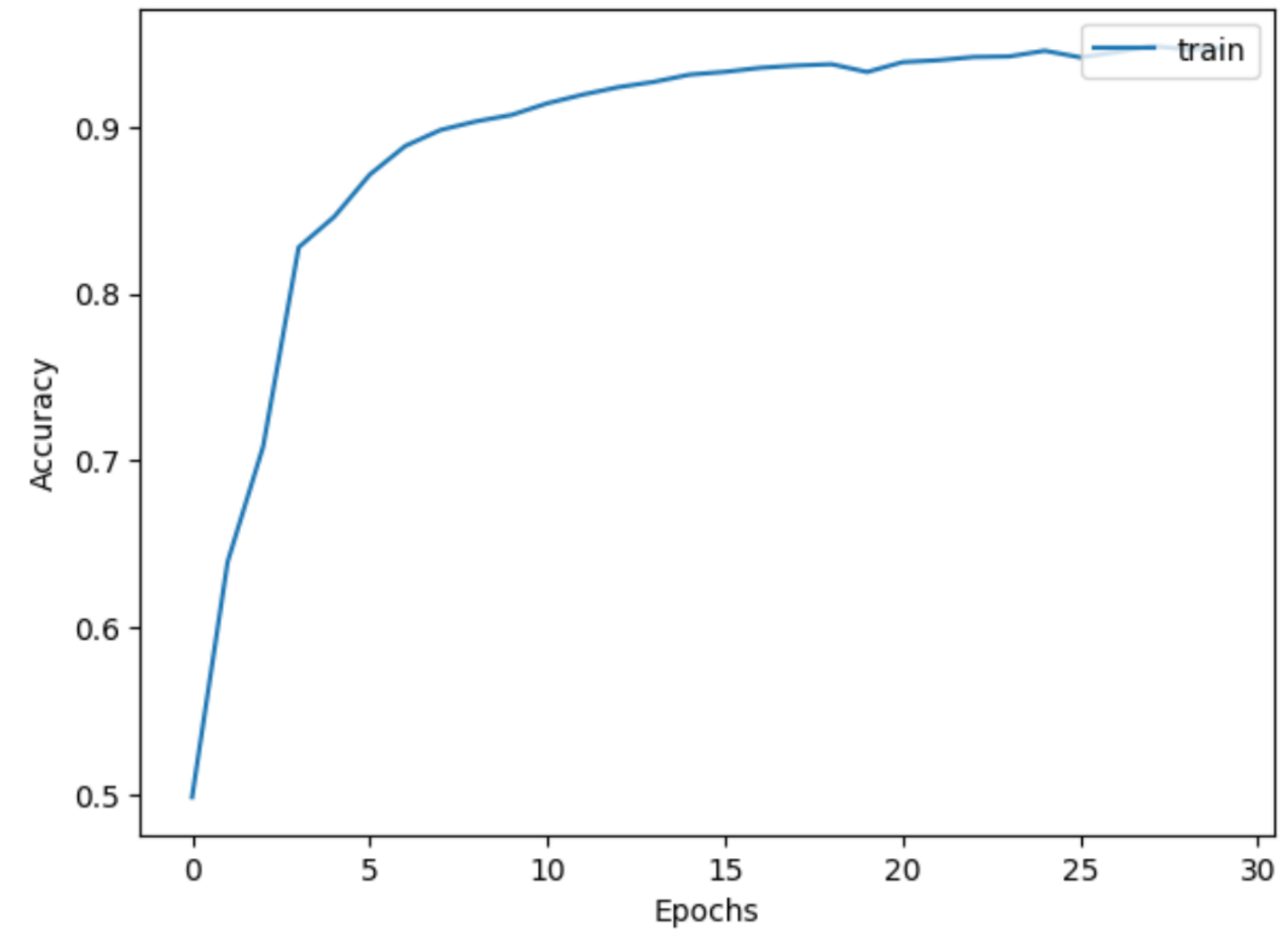
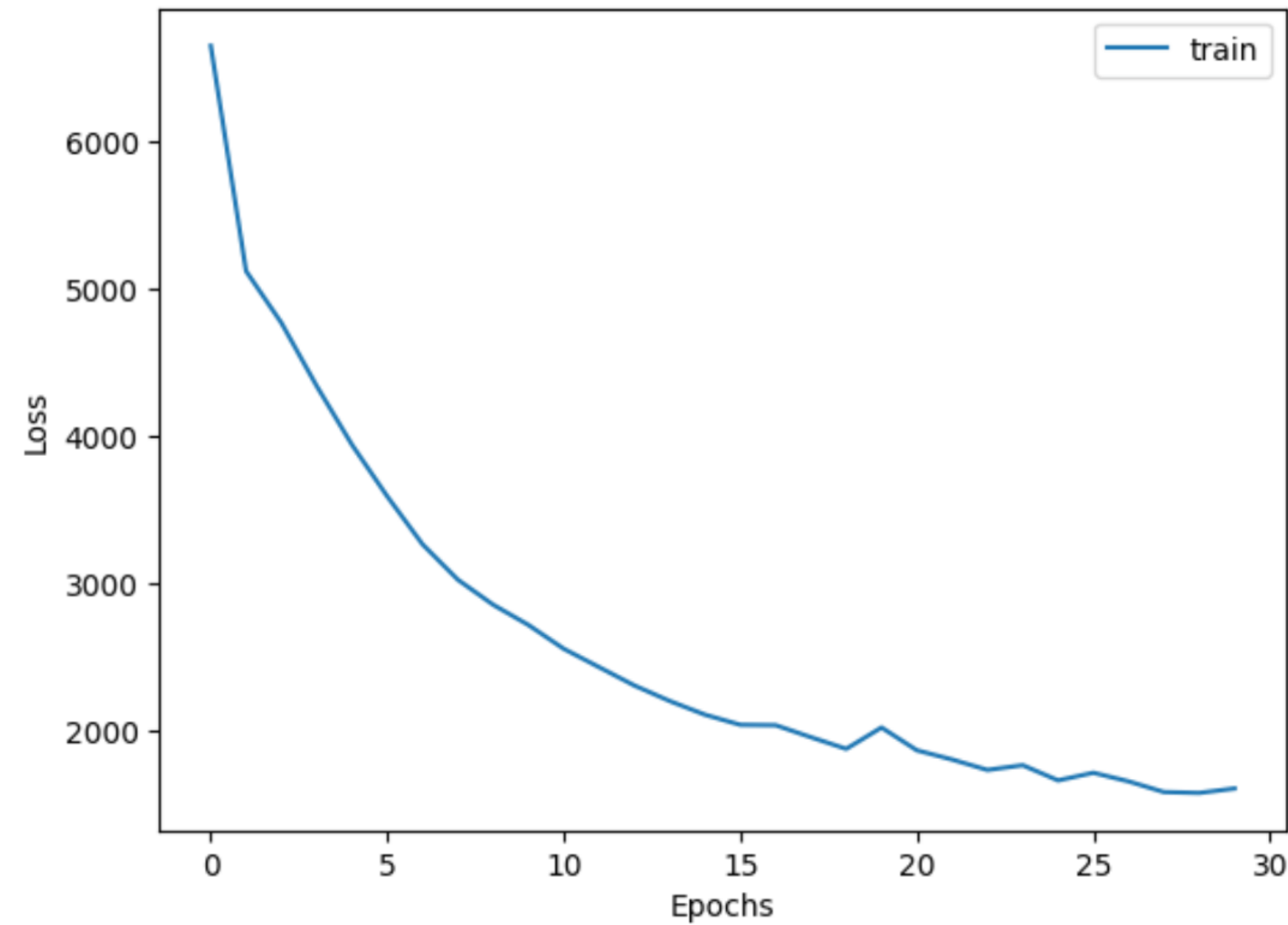
m_{ij} = Invariant mass of pairs

$$m^2 = E^2 - p^2$$

Fully connected graph

Training and testing

Training : Classification of signal and background for $gg \rightarrow b\bar{b}\gamma\gamma$ process



Test :

Event 0:

Node 0: Predicted = 0, Probabilities = [0.9951716 0.00482834]
Node 1: Predicted = 0, Probabilities = [0.9958319 0.0041681]
Node 2: Predicted = 0, Probabilities = [0.9963774 0.00362262]
Node 3: Predicted = 0, Probabilities = [0.9951822 0.00481772]

Event 3:

Node 0: Predicted = 1, Probabilities = [0.04787536 0.9521247]
Node 1: Predicted = 1, Probabilities = [0.14829516 0.85170484]
Node 2: Predicted = 1, Probabilities = [0.04821268 0.9517873]
Node 3: Predicted = 1, Probabilities = [0.14607167 0.8539284]

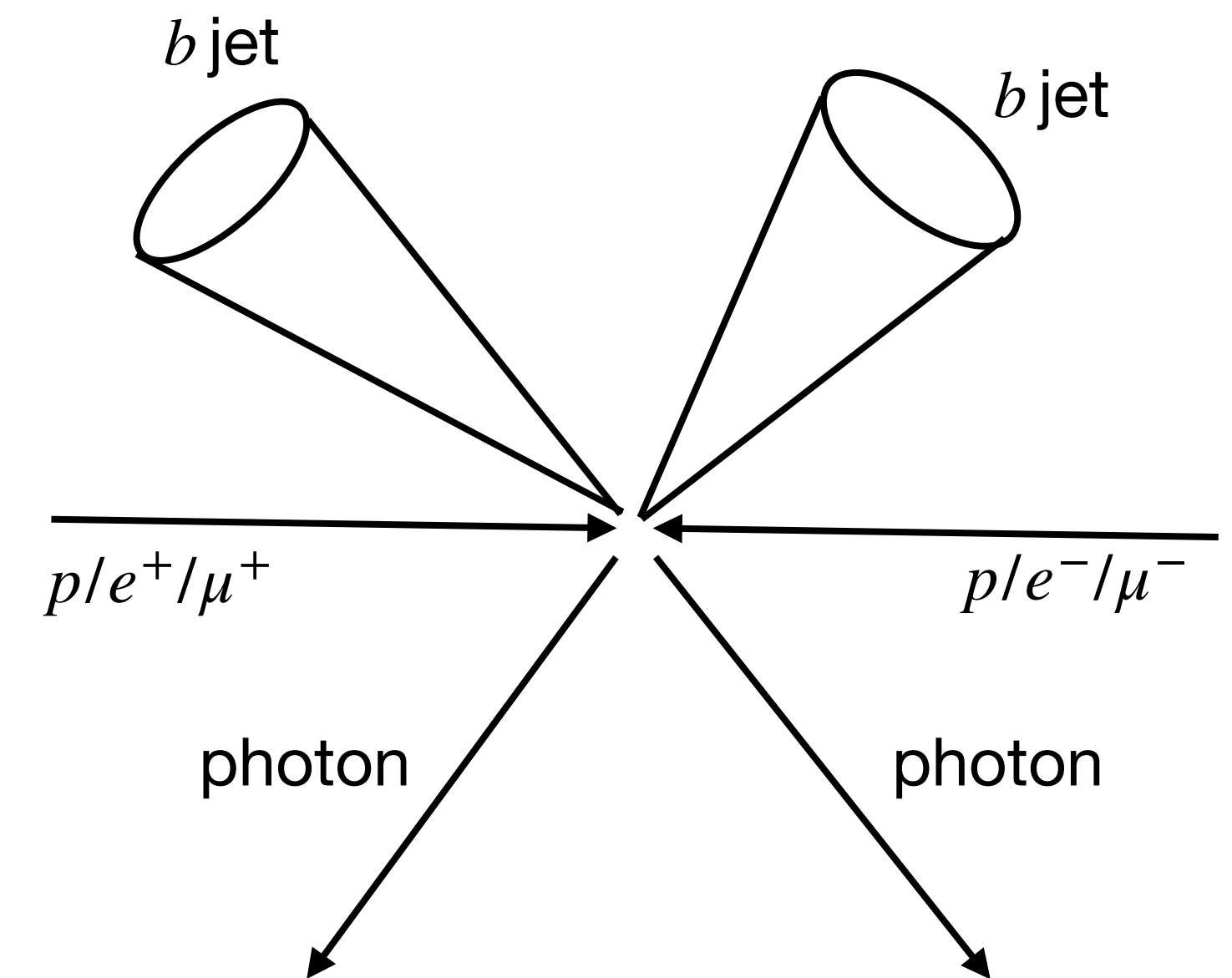
GNN for triple Higgs coupling

- Experimental analysis based cuts and event selection to prepare graph structure

2404.12915

- Events must have two b tagged jets
- Events must have two leading photons with invariant mass $105 \text{ GeV} < m_{\gamma\gamma} < 160 \text{ GeV}$
- Leading photon $p_T > 0.35m_{\gamma\gamma}$ and subleading photon $p_T > 0.25m_{\gamma\gamma}$
- No electrons or muons are present in the event

n	jet1	jet2	γ	γ	MET	pT	eta	phi
1	1	0	0	0	0	62	2.38	3.05
2	0	1	0	0	0	43	0.21	1.73
3	0	0	1	0	0	99	-1.68	-0.39
4	0	0	0	0	0	21	0.05	2.65
5	0	0	0	0	6	0	-2.01	-4.61



$$105 \text{ GeV} < m_{\gamma\gamma} < 160 \text{ GeV}$$

Summary

- Machine learning applications are useful to study triple Higgs coupling
- A good accuracy is achieved for classifying signal and background events using graph convolution networks
- e^+e^- polarized beam can be useful to improve accuracy
- For polarization combination $P_{e^-} = -0.8$ and $P_{e^+} = 0.3$ the accuracy to measure κ is about 57 % at $\mathcal{L} = 2 \text{ ab}^{-1}$
- Sensitivity may be improved by incorporating other channels as well
- A comparative study of triple Higgs coupling for HL hadron, electron-positron and muon colliders will be useful for future direction of particle colliders

Thank you
for your attention

Z-polarization for trilinear coupling

- A_{yz} is odd under naive time reversal

1604.06677 1508.04592

1508.04592 1904.06663

$$A_{yz} \equiv \frac{\sigma(\cos \theta \sin \phi > 0) - \sigma(\cos \theta \sin \phi < 0)}{\sigma(\cos \theta \sin \phi > 0) + \sigma(\cos \theta \sin \phi < 0)} = \frac{2}{\pi} \sqrt{\frac{2}{3}} T_{yz}$$

$T_{yz} \rightarrow$ Polarization component of Z

Can be realised from the transformation properties of $\cos \theta \sin \phi$

- **Naive time reversal:** Reversal of direction of all spins and momenta but not interchange of initial and final state
- CP-even angular asymmetry odd under naive time reversal **is either less sensitive or independent of tree level anomalous couplings**

	A_x	A_y	A_z	A_{xy}	A_{xz}	A_{yz}	A_{zz}	$A_{x^2-y^2}$
T	+1	-1	+1	-1	+1	-1	+1	+1
CP	+1	-1	-1	-1	-1	+1	+1	+1

Requires an absorptive part for non-zero value, CPT theorem!